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Characterising Electoral Systems: An Empirical Application of Aggregated Threshold Functions

RUBÉN RUIZ-RUFINO

This article proposes a new way to measure proportionality using aggregated threshold functions. Electoral systems can be summarised by a single value that shows the necessary share of the total vote to win either one seat or half of the seats in parliament. The article calculates aggregate threshold values for 142 different electoral systems that were used in 525 democratic elections between 1946 and 2000. These results are also contrasted with the most commonly used indices of proportionality and turn out to be both substantively and empirically richer. Aggregated threshold functions provide both students and reformers of electoral systems with a measure based purely on institutional variables that offers an exhaustive summary of the functioning of many electoral systems.

Suppose that an electoral reformer seeks to grant some advantage to large political parties over minor ones. How can she know that, for example, by just modifying the assembly size, the electoral formula or the number of districts in a certain way the desired outcome will be produced? Is it possible to anticipate the mechanical behaviour of an electoral system by just looking at its institutional components? And if so, how proportional is the resulting new electoral system expected to be? While the functioning of electoral systems is a recurrent topic in the literature, no broad consensus exists on how to characterise it (Powell 2000; Milesi-Ferretti *et al.* 2002; Persson and Tabellini 2005). In some cases, such as the literature on ethnic conflict, categorical variables are used to summarise relevant independent variables such as the electoral system (Cohen 1997, Reynal-Querol 2002). These studies use distinctive features of an electoral system, like the electoral formula, as the exclusive criterion to group electoral institutions around different categories of a variable. Other sets of studies using electoral

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systems not only as independent variables (Cox 1997) but also as a dependent variable (Boix 1999; Blais *et al.* 2005) employ empirical measures that produce continuous values. In such cases the discussion is centred around the disproportionality index (DI) (Rae *et al.* 1971) and the effective threshold (Taagepera and Shugart 1989), which arguably stand as the dominant empirical measures used in the literature (Katz 1997).

Admittedly novel at the time, these approaches suffer from a number of limitations. Categorical variables are not capable of producing information about the overall functioning of electoral systems classified under the same category. For instance, the choice of proportional representation (PR) electoral systems as a category arguably blurs the distinction between the number of districts or assembly size when attempting to explain the mechanical performance of electoral institutions. It is difficult to say whether the number of districts, or assembly size, has any effect on the mechanical performance of electoral institutions. On the other hand, the focus on dominant empirical measures faces other problems. The choice of the effective threshold can be justified by the goodness of fit, but it is limited to district-level analysis and only refers to the electoral cost of one seat (Lijphart 1994). The different indexes of disproportionality, while informative and accurate (Rae 1971; Gallagher 1991; Monroe 1994), also suffer from limitations such as their dependency on election results. In this sense, it is hard to distinguish between the mechanical and the psychological effects of an electoral system (Duverger 1954) by just looking at the election results.

Recent research in the literature on electoral systems (Taagepera 2007) has allowed for the development of new measures that aim to overcome some of the above-mentioned limitations. Among them, aggregated threshold functions (ATF) constitute an attempt to offer a distinct view of the mechanical workings of electoral systems. Briefly, these functions offer the necessary and sufficient shares of votes nationwide to win any number of seats given the institutional components of an electoral system. Thus defined, aggregated threshold functions summarise the process of converting ballots into seats in each electoral system into unique informative values.¹ This article seeks to go beyond the theoretical approach offered by earlier research by using aggregated threshold functions to produce an empirical measure of the mechanical performance of some electoral systems as a basis for a new way of characterising them. It will also show how these ATF values can be reconciled with actual election results.² The article is structured along three main lines.

The next section briefly reviews how values from ATF are obtained as well as their substantive meaning. A new definition of seat-winning parties is introduced which adds to the theoretical robustness of such functions. The introduction of this measure constitutes a development with respect to earlier research. Initially distinguished by its properties of quasi-universality and generality, plugging the new predictor of the number of seat-winning parties into ATF adds a new distinguishing property: independence from electoral results. Taken together, these properties make aggregated

threshold values interesting to both students of electoral systems and electoral reformers. The former will find in such functions a straightforward analytical tool to anticipate some mechanical behaviour³ of certain electoral systems by simply looking at the seat allocation. In this sense, aggregated threshold functions allow researchers to anticipate some of the political consequences generated by electoral institutions, such as the permeability of the system to the entrance of small parties or the possibility of forming single-party cabinets. Similarly, electoral reformers may find in these functions a tool for forecasting the mechanical performance to be expected from a future electoral system; thus, electoral engineers and other electoral practitioners will be provided with a measure that will help them in designing and testing the effects of new electoral institutions.

In the following section, values generated by aggregated threshold functions are calculated when the total number of seats is one and when the number of total seats equals exactly half of the parliament. These values are calculated for 142 single-seat district and PR electoral systems used in 525 parliamentary democratic elections held around the world between 1946 and 2000. The advantage of using these two values is that a metrical characterisation of the mechanical behaviour of the electoral systems under study can be built. In this sense, the characterisation of electoral systems using the ATF values generated from half of the seats in the parliament will prove useful to draw some interesting conclusions about the process of transforming votes into seats.⁴

The article goes on to contrast the values obtained from the ATF with other empirical measures such as various disproportionality indexes and the effective threshold. Three main results are presented. Firstly, the values generated by aggregated threshold functions may overcome the strong dependency on election results that disproportionality indexes entail. ATF thus summarise the mechanical behaviour of most electoral systems without relying on any electoral results. Secondly, aggregated threshold functions have a clear substantive meaning that makes them distinguishable from other empirical measures such as the effective threshold. While the effective threshold may be understood as a 'range of possibilities' (Boix 1999: 614), aggregated threshold functions precisely indicate the minimum value required to win either one seat or half of the seats in the parliament. Finally, the article shows how the values generated by the aggregated threshold functions anticipate the major trends actually observed in the process of transforming votes into seats in 15 democracies.

Aggregated Threshold Functions

Definition

Aggregated threshold functions allow anyone interested in electoral institutions to anticipate the mechanical behaviour of an electoral system.

To do that, ATF are defined for some complete electoral systems. A complete electoral system is an institution consisting of the following elements: an electoral formula, a vector that contains the distribution of all district magnitudes into which the country is divided, an integer that shows the total number of districts and the number of seats in the legislative assembly.⁵ As a consequence of this institutional setting, a number of parties emerge. These parties compete among each other in every election in order to win as many votes as possible. Given that aggregated threshold functions provide values that summarise the transformation of votes into seats, political parties are consequently incorporated into the functions. It is also assumed that all district magnitudes show a perfect relation with their voting population. In other words, no malapportionment effect is taken into account.

With all these elements, aggregated threshold functions are conceived as a set of functions that calculate the necessary and sufficient share of the total vote to win a given number of seats that are particularly distributed among all district magnitudes given any electoral formula, any number of districts, any legislative assembly size and, finally, given any number of parties.⁶

Thus defined, ATF have two interesting properties. Aggregated threshold functions are general and quasi-universal. Generality refers to the capacity to summarise the mechanical functioning of any electoral system taking into account all districts, and not just one, into which a country is divided. This property allows us, therefore, to observe the global mechanical functioning of particular institutional designs.⁷ Quasi-universality means that these functions can be applied to a large number of electoral systems.⁸

The Number of Seat-Winning Parties

The inclusion of political parties in the aggregated threshold functions requires some clarification. One of the earliest results in electoral studies is that party systems are conditioned by the institutional components of the electoral rules (Rae 1971; Cox 1997). Recent studies have further demonstrated that the number of parties can be calculated approximately using some institutional components of an electoral system. As suggested by Taagepera (2007), given an assembly size, S , and a district magnitude, M_d , the best guess to anticipate the number of seat-winning parties can be made using the following formula:⁹

$$P = (S * M_d)^{1/4} \quad (1)$$

Since generality is a distinctive feature of aggregated threshold functions, M_d above can be substituted by the average district magnitude¹⁰ in order to obtain the number of parties that could win seats in the assembly nationwide. After this substitution, the number of potentially seat-winning parties is¹¹

$$P = \left(\frac{S^2}{E} \right)^{1/4} \quad (2)$$

Since aggregated threshold functions refer to the conditions required to win a given number of seats, the number of seat-winning parties is the appropriate indicator that should be plugged into the functions. The number of competing parties could be plugged instead but this predictor, though interesting, does not match with the substantive meaning of ATF. Normally, the number of competing parties in an election is high and ranges from large to rather small parties. A measure that establishes the electoral threshold required to win a given number of seats does not consider small parties but, rather, those parties with the capacity to win at least one seat. In other words, a predictor that calculates the number of seat-winning parties, like the one proposed by Taagepera (2007), must be incorporated into the ATF.¹² The inclusion of seat-winning parties in ATF is subject to a last condition that is required for simplicity. It is assumed that the number of seat-winning parties is the same in all districts.

Finally, by calculating the number of seat-winning parties with just institutional variables, a third property of ATF emerges: independence. Independence means that one can estimate the mechanical effects of an electoral system without relying on the electoral results it generates as, for example, disproportionality indexes do.

Necessary vs. Sufficient Values

By definition, aggregated threshold functions calculate *ex ante* the necessary and sufficient shares of the total vote required to win a given number of seats based on purely institutional variables.¹³ To use aggregated threshold functions as a tool to characterise electoral systems, a further conceptual refinement is required. It must be decided whether to opt for the aggregated threshold function of necessary votes or that of sufficient votes. The choice between these two functions is not just a question of taste. For any measure to be parsimonious, it must combine simplicity and explanatory power. So, the simpler and the greater the substantive meaning of any measure, the more parsimonious it will be. Therefore, throughout, this article only uses the ATF of necessary votes for the following reasons. First, by using this function I am adopting an exclusive criterion: any party that obtains a share of the total vote below the necessary condition will have no chance of winning the total number of seats for which the function was applied. Second, the aggregated threshold function of necessary votes decreases the uncertainty about the total number of seats that can be won implicit in the function of sufficient votes. Whereas the aggregated threshold function of necessary votes sets the threshold that must be reached to win a concrete number of total seats, the function of sufficient votes leaves open the possibility of winning a higher number.

This necessary condition allows electoral systems to be characterised by two different values. First, electoral systems can be characterised by the minimum share of the total vote that a party would require to win just one

seat.¹⁴ This value could be used as a measure to test the flexibility of any electoral system regarding the entry of minor parties into the legislative assembly. If the minimum value required to win one seat in the parliament is too low then a much more fragmented parliament could be expected than in the case of a higher value. Studies on the emergence of extremist parties (Veugelers and Magnan 2005; Abedi 2002) or on ethnic parties' capacity for success in heterogeneous societies (Moser 1999) could benefit from this measure.

A second and even more appealing criterion for characterising electoral systems is the minimum share of the total vote needed to win half of the seats in the legislative assembly. This value would allow for the characterisation of complete electoral systems by locating them on a continuum and comparing them with an ideal point. If we define this point as perfect proportionality,¹⁵ we can have a continuous value that measures the distance between the minimum share of the total vote required by a party to win a majority of seats in the legislative assembly and this ideal point. In other words, we are calculating the immediate mechanical response of an electoral system in which half of the seats in the legislature are allocated in an easy scenario, that is, where the cost of winning such seats is cheapest for any political party.

Imagine that ATF predicts for country *X* a value of 0.3 as the minimum share of the total vote necessary to win half of the seats in the parliament. This value is, actually, saying that without taking into account the rational calculations of the voters, a party can win 50 per cent of the seats with just 30 per cent of the total vote. In other words, the mechanical functioning of such an electoral system is far from being perfectly proportional. This approach will then help us understand, for example, the propensity of a complete electoral system to produce coalition governments in parliamentary democracies. The higher the minimum share of the total vote needed to win half of the seats in the legislative assembly, the less likely it will be to find single-party ruling coalitions in parliaments (King *et al.* 1990; Lupia and Strom 1995). Another case in which this value would be helpful is the analysis of the role that electoral systems have played in transforming democracies (Birch 2003).

Data and Methodology

In this study, aggregated threshold functions are applied to 142 different complete electoral systems that were used in 525 parliamentary elections occurring between 1946 and 2000 under either a *single-seat district* or a PR electoral system. The data have been collected from Golder (2005) and have been expanded by incorporating district data in those cases where such information was available. Given that aggregated threshold functions do not rely on electoral results, some electoral systems cannot be characterised using ATF.¹⁶ In any case, the data presented here cover about 61 per cent of the elections taking place in the world between 1946 and 2000.¹⁷

Table 1 shows the total number of elections and electoral systems used in this article. From this table we can see that *single-seat district* electoral systems and proportional representation electoral systems were adopted for about the same number of elections. *Single-seat district* electoral systems were used for 264 elections whereas 261 elections occurred under proportional representation systems.

Finally, given that the unit of analysis is the complete electoral system, two electoral systems will be considered different if any of the following criteria are observed:

1. A change of electoral formula.
2. A change in the number of districts into which the country is divided.
3. A change in the number of seats in the legislative assembly. A change is considered to have taken place when there has been a change of 20 per cent in the size of the legislature (Lijphart 1994).
4. An electoral system established after a period of dictatorship is also considered to be a new electoral system (Golder 2005).

Aggregated threshold functions are applied considering a particular distribution of seats among all districts. That distribution of seats minimises the value of the function and therefore offers the values we are interested in: the minimum share of the total vote below which it is impossible to win either one seat or half of the seats in the legislative assembly. In *single-seat district* electoral systems, this value is a function of assembly size and the number of parties (if ATF refers to one seat) or just the number of parties (if ATF refers to half of the seats in parliament). In the case of both quota-based and divisor-based electoral systems, that distribution of seats is obtained by allocating all seats in the smallest districts. For example, in the case of the minimum share of the total vote necessary to win just one seat, aggregated threshold functions are applied to the smallest district in the country.¹⁸

Note, however, that for those cases where the distribution of district magnitudes is not known, the average district magnitude can be used instead. The correlation between the values obtained using the distribution of districts and the average district magnitude is over 0.85, which makes average district magnitude a good proxy of ATF.¹⁹ In the Appendix, interested readers can find the mathematical form of the aggregated

TABLE 1
NUMBER OF ELECTIONS AND ELECTORAL SYSTEMS

Type of electoral system	Elections	Electoral systems
<i>Single-seat district</i>	264	59
PR-divisors	170	54
PR-quota	91	29
Total	525	142

threshold functions as well as the abbreviated function using the average district magnitude.

The Characterisation of Electoral Systems

Single-Seat District Electoral Systems

Aggregated threshold functions have been applied to complete electoral systems that use the first-past-the-post (FPTP), block vote (BV), party block (PV) and the two-rounds system (TRS). As mentioned above, these electoral systems allow the application of the functions regardless of the distribution of the vote. It should be noted, however, that in the case of the TRS, aggregated threshold functions are applied to the first round only.²⁰

Table 2 summarises the values obtained in these *single-seat district* electoral systems. The first column shows descriptive data when the functions are applied to just one seat; the second column shows the same information when aggregated threshold functions are applied to half of the seats in parliament.²¹

The values shown in Table 2 illustrate well the majoritarian nature of these electoral systems. Consider first the values obtained when the number of seats equals one. The minimum value is produced by the electoral system used between 1958 and 1981 in the French legislative elections. Here, a TRS electoral formula was used to distribute an average of 470 seats between approximately six parties. The minimum threshold to win a seat under this institutional setting is about 0.04 per cent of the total vote. The maximum value was obtained in the electoral system used in the 1984 legislative election in St. Kitts and Nevis, where 11 seats were chosen using FPTP and where 1.82 parties competed in the electoral contest.²² Under this electoral system, the minimum share of the total vote needed to win a seat was about 3.71 per cent.

When district magnitude equals 1, the minimum share necessary to win the majority of seats in the parliament depends exclusively on the number of seat-winning parties. For example, given a complete electoral system with a FPTP electoral formula, if the number of parties competing and winning the total number of seats is two, then the share of votes below which a party cannot win half of the seats in parliament is 25 per cent. This situation arises

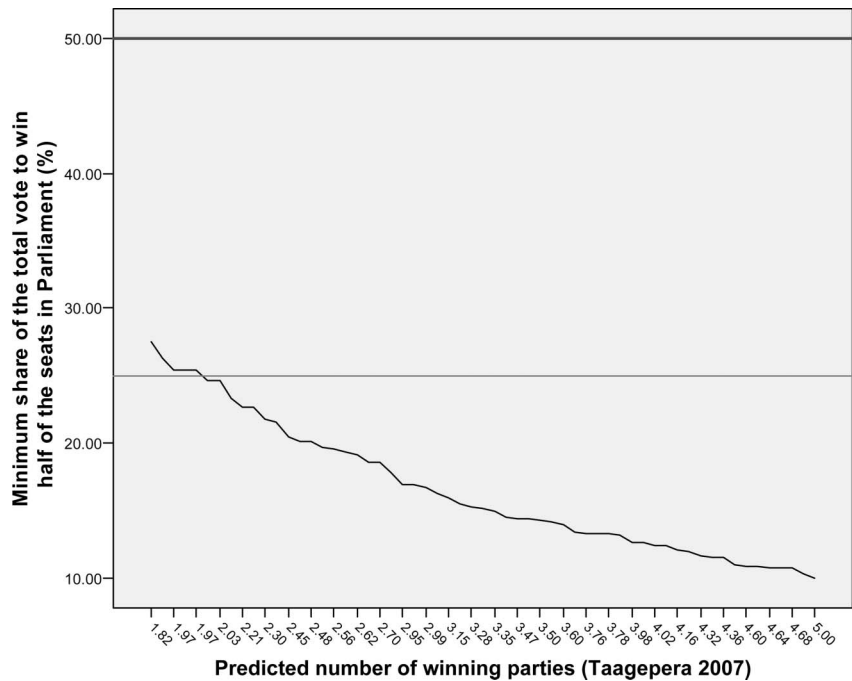
TABLE 2
DESCRIPTIVE VALUES FOR WINNER-TAKES-ALL ELECTORAL SYSTEMS

	1 seat	Half seats in parliament
Mean (%)	0.94	16.5
Std dev. (%)	1.1	4.9
Minimum (%)	0.04	10
Maximum (%)	3.71	27.46

when such a number of seats is won in half of the districts with the minimum number of votes, and no votes are won in the remaining districts.²³ This is, of course, an extreme case but it is useful because it gives us the value below which it is impossible to win half of the seats in parliament under any circumstances. The number of seat-winning parties is, then, inversely proportional to the minimum number of votes required to win the majority of the seats in the parliament. The idea is well illustrated by looking at the solid decreasing line in Figure 1.²⁴

In terms of perfect proportionality, these values show exactly how disproportionate a *single-seat district* electoral system can be. If perfect proportionality is understood as implying that a party's share of the vote equals the share of seats it obtains, then the mechanical transformation of votes into seats of the electoral systems analysed here are far from being proportional. The thick horizontal line in Figure 1 shows where the point of perfect proportionality lies. Each electoral system can, therefore, be observed in relation to this ideal point. As Table 2 shows, on average the minimum value needed to win half of the seats in the legislative assembly is about 16.5 per cent of the total vote share. That is an average distance of 33.5 per cent away from the ideal point of perfect proportionality.

FIGURE 1
MINIMUM SHARE OF THE TOTAL VOTE NEEDED TO WIN HALF OF THE SEATS IN
PARLIAMENT FOR WINNER-TAKES-ALL ELECTORAL SYSTEMS



PR Electoral Systems

I turn now to analysing aggregated threshold values for those electoral systems that use a proportional representation electoral formula. These electoral systems normally use multi-member districts, and depending on the method used to distribute seats one can distinguish between divisor-based or quota-based proportional representation electoral systems. Both divisor- and quota-based electoral formulas are mostly used in party-list systems. In fact, 47 countries used this type of electoral formula in 261 parliamentary elections between 1946 and 2000.

Briefly, quota-based electoral systems distribute seats using a predetermined quota calculated using the district magnitude. Parties win as many seats as full quotas obtained and the remaining seats are allocated using procedures like the largest remainder method.²⁵ A total of 29 quota-based electoral systems were used in 91 elections between 1946 and 2000.

On the other hand, in divisor-based electoral systems a divisor must be found to enable the calculation of the averages needed in order to distribute the seats allocated in each district (Balinski and Young 1982; Lijphart 1994). Divisor-based electoral formulas were used in 170 general elections with 54 different complete electoral systems.

Table 3 shows a summary of the values calculated for PR electoral systems. Looking at the values calculated for one seat, some remarks can be made. First, as the literature has already shown (Rae 1971; Gallagher 1992), the average is lower in quota-based than in divisor-based electoral systems. The price of winning a seat is cheaper in the former type of system. Second, despite the difference just noted, the minimum values necessary to win a seat are close under both types of electoral system and they approach the values shown for single-seat district electoral systems. The reason for this is that some PR electoral systems, where seats are allocated in multi-member districts according to some population criterion, also incorporate single-member districts where the effect of PR formulae just does not apply. This is the case, for example, for the electoral system used in Brazil in 1962 or 1998 or the electoral system used in Spain since 1977. Third, the values needed to win one seat must be interpreted with some caution because they may not show the real 'price' of winning a seat. As mentioned above, legal thresholds should be taken into consideration at this stage.²⁶

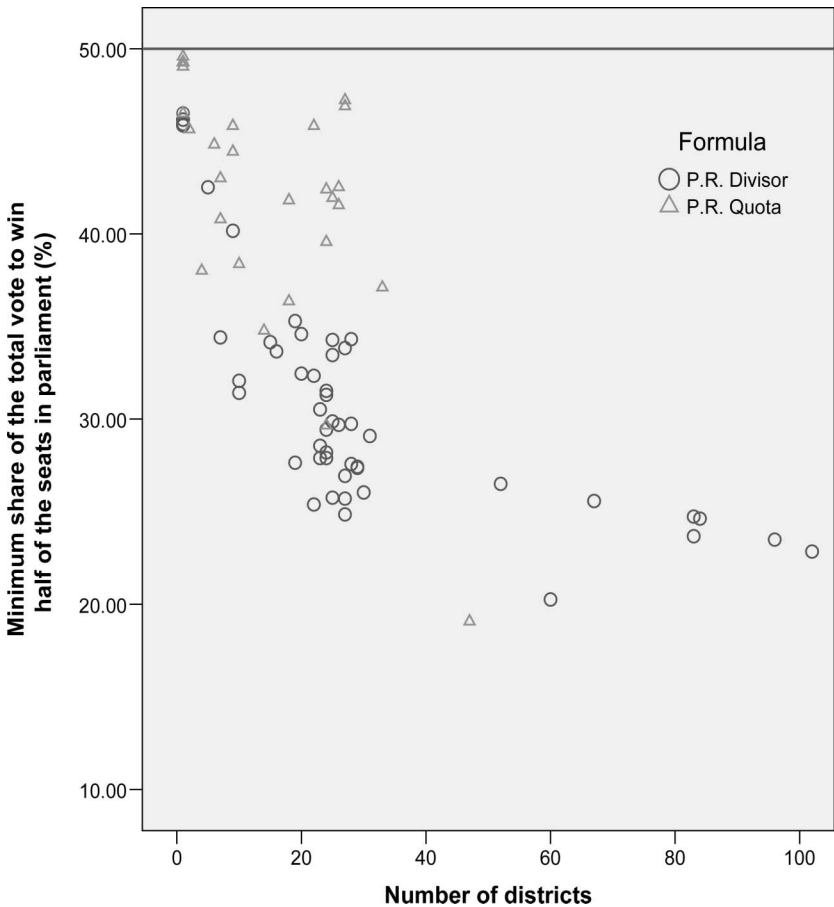
TABLE 3
DESCRIPTIVE VALUES FOR PR COMPLETE ELECTORAL SYSTEMS

	PR quota		PR divisor	
	1 seat	Half seats in parliament	1 seat	Half seats in parliament
Mean (%)	0.20	41.9	0.31	31.42
Std dev. (%)	0.16	6.7	0.29	6.78
Minimum (%)	0.1	19.08	0.2	20.26
Maximum (%)	0.53	49.6	0.92	46.51

Similar observations about the type of electoral formula can be made if we look at the values required to win half of the seats in the parliament. On average, if a complete electoral system uses a quota-based formula, then the minimum value required to win half of the seats in parliament is about 42 per cent whereas if a divisor-based electoral formula is used that value is about 31.5 per cent. If electoral formulas explain the proportionality of the system, the number of districts also seems to be important: in both types of electoral systems, higher values are obtained in electoral systems with just a single district.²⁷ These ideas can be better explained by looking at Figure 2.

Locating all complete electoral systems along a continuum, and contrasting their minimum thresholds for winning half of the seats in parliament with the point of perfect proportionality, as Figure 2 shows, has

FIGURE 2
PR ELECTORAL SYSTEMS IN RELATION TO THE POINT OF
PERFECT PROPORTIONALITY



graphical advantages. First, we can draw some distinctions depending on the type of electoral formula that is being used. In this sense, PR electoral systems that use any type of quota-based formula produce results closer to the line of perfect proportionality than those electoral systems that use any of the divisor-based electoral formulas. Practically all quota-based electoral systems that have been characterised here use the Hare quota as the electoral formula for allocating seats.²⁸ This electoral formula produces more proportional results than other formulas, such as the d'Hondt formula, which is used in most divisor-based electoral systems (Farrell 2001).

Second, and equally interesting, is that the 'proportional' label traditionally attached to these electoral systems may be challenged if one focuses on the values produced by aggregated threshold functions. As Figure 2 shows, the number of districts is important when calculating the minimum number of votes required to obtain a majority of seats in the parliament. In fact, holding all other institutional variables constant, when the number of districts increases, the minimum share of the total vote for winning half of the seats in the legislature decreases, i.e. the electoral system becomes less proportional. This is in line with earlier studies which have found that district magnitude and proportionality are positively correlated (Taagepera and Shugart 1989; Lijphart 1994). The finding here strengthens this conclusion by testing it not at a micro-level (the district) but at a macro-level (the whole electoral territory). Therefore, in considering the mechanical functioning of a complete proportional representation electoral system as a whole, the number of districts is important, which means that some so-called 'proportional' electoral systems may not be producing actual proportional results. In other words, the higher the number of districts, the further away from the line of perfect proportionality.

Consider, for example, the case of the Dominican Republic. Since 1966 this Caribbean country has held periodic competitive elections using a PR electoral system where 120 seats are elected in 30 districts using a D'Hondt electoral formula. On average the ATF predicts that about 26 per cent is the minimum share of the total vote needed to win half of the seats in the parliament. This is a value that truly challenges the 'proportional' label attached to the Dominican electoral system. In this sense, the data used here show that about 10 per cent of the PR cases fall below the 25 per cent line that was established for *single-seat district* electoral systems.²⁹ This is clearly an advantage of using ATF: aggregated threshold functions provide a picture of how the electoral system works as a whole. By looking at graphs like the one in Figure 2, one can grade the proportionality generated by the process of converting votes into seats of either a *single-seat district* or a PR electoral system.

Aggregated Threshold Functions and Other Electoral Indices in Comparative Perspective

The literature on electoral systems offers not only a variety of different indices for measuring the performance of electoral systems but also

interesting discussions about them (Gallagher 1991; Monroe 1994). Do the values generated by the aggregated threshold functions offer any different information? In other words, are the ATF values richer in any sense than other existing electoral indices? The purpose of this section is to show that while other indices summarise accurately the behaviour of both electoral institutions and electorates, ATF can be useful to anticipate only purely mechanical responses generated only by electoral institutions.

This empirical exercise starts by calculating some of the major indices used in the literature and contrasting them with the values produced by aggregated threshold functions. The data used here correspond to the most recent election in 13 European countries plus Canada and the United States existing in the database used in this article. Since the main purpose of this section is to just describe and to compare the outcomes generated by different electoral indices including ATF, the selection of countries intends to show the different types of electoral systems covered in the database analysed here. So, there are four *single-seat district* electoral systems using FPTP and TRS, eight PR divisor-based electoral systems using D'Hondt, Sainte-Laguë and Mod. Sainte-Laguë and three PR quota-based electoral systems using the Droop quota. The indices calculated are Loosemore and Hanby's (L-H), Rae's, Gallagher's least squares and Sainte-Laguë (S-L) indices, and the effective threshold. Data are presented in Table 4.

Michael Gallagher is certainly right when he observes that indices based on largest remainders, such as the Loosemore and Hanby index (1971), do not survive paradoxes like the so-called 'new state' paradox.³⁰ Gallagher's (2005) defence of the least-squares method is convincing since it is a method that is not vulnerable to such paradoxes, and this index does really reflect the mechanics inside an electoral system. However, Gallagher's

TABLE 4
AGGREGATED THRESHOLD VALUES AGAINST OTHER INDICES

Country	Year	L-H	Rae	Least squares	S-L	Effec. thresh.	ATF
UK	1997	0.212	0.038	0.168	0.223	0.375	0.100
France	1997	0.304	0.060	0.186	0.438	0.375	0.103
USA	1998	0.040	0.039	0.033	0.033	0.375	0.110
Canada	2000	0.171	0.068	0.137	0.151	0.375	0.124
Sweden	1968	0.037	0.014	0.029	0.014	0.081	0.343
Norway	1985	0.065	0.021	0.042	0.032	0.082	0.353
Bulgaria	1997	0.040	0.016	0.039	0.008	0.086	0.241
Switzerland	1999	0.042	0.010	0.028	0.011	0.087	0.297
Portugal	1999	0.069	0.027	0.051	0.027	0.061	0.325
Finland	1999	0.059	0.013	0.031	0.020	0.052	0.342
Spain	2000	0.076	0.012	0.061	0.045	0.097	0.265
Netherlands	1998	0.017	0.003	0.011	0.001	0.005	0.465
Latvia	1998	0.054	0.053	0.054	0.054	0.054	0.425
Slovak Rep.	1998	0.029	0.009	0.019	0.004	0.005	0.465
Luxembourg	1999	0.056	0.018	0.035	0.022	0.054	0.380

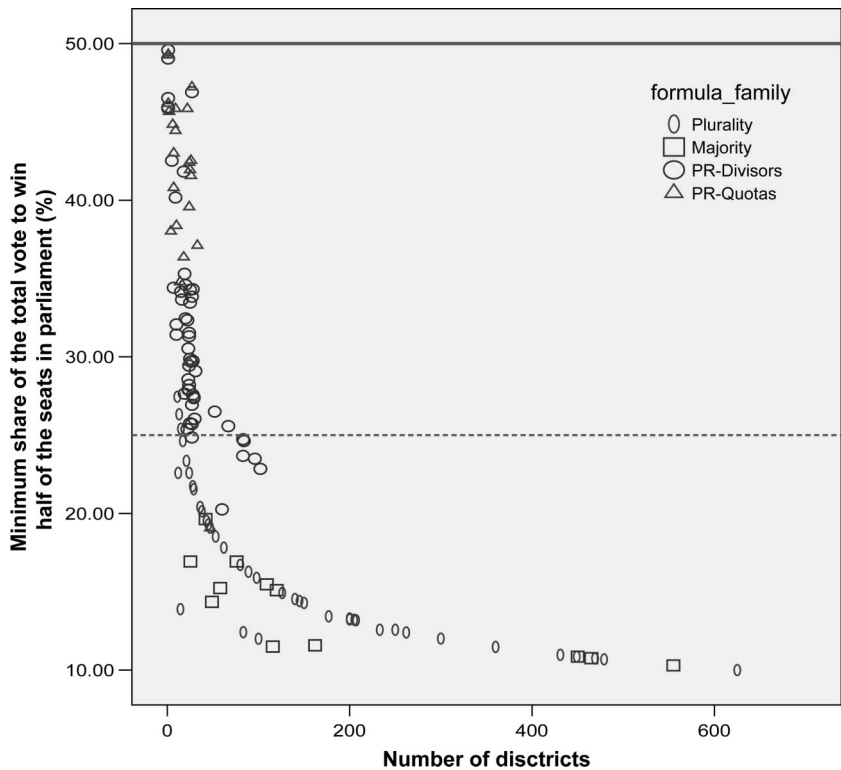
Sources: Greece (Clogg 1992) Election Results Archive: <http://www.binghamton.edu/cdp/era/index.html>; Inter-Parliamentary Union: <http://www.ipu.org/english/home.htm>

least-squares index has two limitations that are also found in Loosemore and Hanby's index as well as in Rae's. First, and most important, all of them rely on the occurrence of the election. This means that these indices, as well as the Sainte-Laguë index, measure the effect of the electoral system on each individual election. The virtue of these indicators is that they offer a fair portrait of the relationship between electoral institutions and voters and they accurately summarise this information into a single value for each election. What this index cannot do, however, is show the raw effect of the institutional components of the electoral systems. This is a small but significant nuance since, for instance, electoral reformers could not anticipate the possible outcome of an electoral system that has not been put into practice yet. Aggregated threshold functions are not conditioned by such a problem. Instead, they anticipate the raw mechanical functioning of any electoral system without relying on the occurrence of an election. By calculating the necessary share of the total vote to win a given number of seats, ATF provides electoral reformers with a tool that can be used to forecast possible political consequences of a potentially new electoral system.

Second, these four indices – L-H, Rae's, least-squares, and S-L – fail to provide a clear substantive interpretation of what they are measuring.³¹ How should we interpret the differences between the electoral systems used in Bulgaria in 1997 and in the Netherlands in 1998 if we look at the L-H or least-squares indices? Both indices produce a value of approximately 0.04 for the case of Bulgaria and 0.017 (L-H) or 0.011 (least-squares) for the Netherlands in 1998. It is true that, according to these indices, the election in Bulgaria in 1997 was more disproportional than the one in the Netherlands, but the question is: by how much? The same can be said of the Sainte-Laguë index: by looking at the electoral systems used in Finland and the Slovak Republic we see that the S-L index produces values of 0.02 and 0.004 respectively so we know that the Slovak electoral system is more proportional than the Finnish one. After looking at this data, can we infer that the electoral system used in the Slovak Republic is five times more proportional than the one used in Finland? In this sense, ATF values are more informative than the above-mentioned electoral indices. By determining the minimum share of the total vote necessary to win half of the seats in parliament, aggregated threshold functions introduce a more intuitive approach based on a comparison with the point of perfect proportionality. Going back to the systems used in the Slovak Republic and Finland, we can say that the former is almost perfectly proportional whereas the distance of the latter from the point of perfect proportionality is much greater (approximately 0.16 points). The fact that in the Slovak Republic there is only one district and in Finland there are 15 is one key explanation of these differences.

In sum, as Figure 3 shows, the contrast between the values produced by aggregated threshold functions and the point of perfect proportionality provides students of electoral systems with a straightforward informative

FIGURE 3
PROXIMITY OF ELECTORAL SYSTEMS TO THE LINE OF
PERFECT PROPORTIONALITY



measure of the distance of each system from perfect proportionality. This metric can be used to rank all electoral systems according to their proximity to the line of perfect proportionality.

Finally, the effective threshold functions calculated and developed primarily by Lijphart (1994) and Taagepera and Shugart (1989), which are shown in the seventh column of Table 4, also have some limitations. The effective threshold only refers to the cost of a seat in a given district magnitude, focusing exclusively on how electoral systems behave at district level. In contrast, aggregated threshold functions provide information about the necessary conditions for winning any number of seats in any complete electoral system. ATF has, then, a broader scope than the effective threshold. Effective thresholds, calculated as the mean of the threshold of representation and the threshold of exclusion, also fail to provide a precise meaning for what is being measured; Carles Boix (1999) refers to this value not as a concrete number but as a ‘range of possibilities’ for obtaining representation. As both Table 4 and Figure 3 show, the data produced by

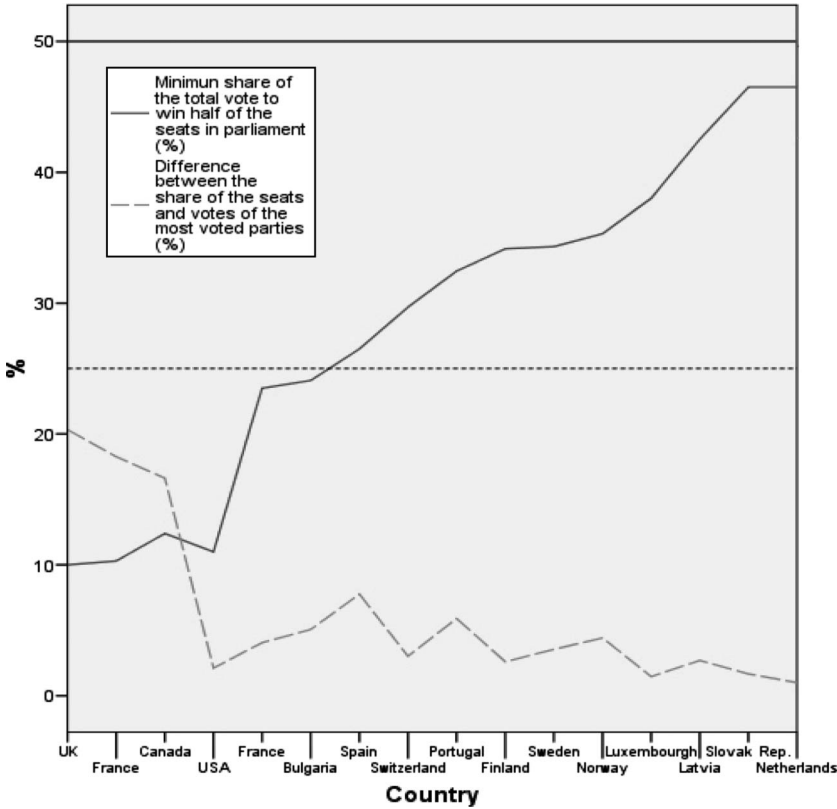
aggregated threshold functions do have a clear meaning of their own. Namely, the minimum total vote share necessary to win a majority of seats in the parliament or the quantitative distance of a particular electoral system vis-à-vis the point of perfect proportionality.

A final remark must be made at this point. As I have already said, aggregated threshold functions summarise electoral systems around the minimum value needed to win half of the seats in a parliament. We know, however, that these values are rarely observed. For example, ATF predicts that no party will win a bare majority of seats in the House of Commons unless a minimum of 10 per cent of the national vote is obtained. Yet we know that majorities of the seats in that House are obtained by a much higher percentage of votes – higher even than 25 per cent of the vote. Likewise, ATF predicts that a minimum of 26.5 per cent of the total vote is required to win half of the seats in the Spanish parliament and yet we know that the last party to win an overwhelming majority in that house – the conservative party (PP) in 2000 – did so after winning 44.52 per cent of the total vote. The question, then, is why we should use the values generated by aggregated threshold functions if the empirical results may be far away from them. Figure 4 plots aggregated threshold values and the differences between the seat share and vote share for the most successful parties for all the countries in Table 4.

The difference between the seat share and the vote share of the most successful party can be used as a measure for observing to what extent electoral systems reward big parties. The greater the difference, the more disproportional is the electoral system. Figure 4 illustrates this idea. The dashed line shows the seat–vote difference and, not surprisingly, the values corresponding to the UK, France and Canada are the highest as well as being above those obtained by aggregated threshold functions. These countries use majoritarian electoral systems, which is why the aggregated threshold values represented by the solid line are far below not only the ideal 50 per cent but also the 25 per cent limit (dotted line) established for *single-seat district* electoral systems. So the empirical results produced by these electoral systems display the majoritarian nature that was anticipated by ATF.

Furthermore, Figure 4 shows some other interesting things. For example, some ‘proportional’ countries are not as ‘proportional’ as the literature would characterise them. Spain and Portugal illustrate the idea fairly well.³² ATFs anticipated the majoritarian bias of the Spanish electoral system by producing a value of 26.5 per cent as the minimum share of the total vote needed to win half of the seats in parliament, and the empirical results show that the seat–vote difference is about 8 per cent; likewise, aggregated threshold functions predicted a minimum value of 32.5 per cent for Portugal and the seat–vote difference for the 1999 parliamentary election was about 6 per cent for the most-voted-for party. Furthermore, those countries with the smallest seat–vote difference, the Slovak Republic (1.7 per cent) and the

FIGURE 4
AGGREGATED THRESHOLD VALUES AND THE DIFFERENCE BETWEEN THE
SHARE OF SEATS AND VOTES OF THE MOST-VOTED-FOR PARTY



Netherlands (1 per cent), are also closer to perfect proportionality according to aggregated threshold functions, as one can see by looking at how close the solid line is from the 50 per cent value.

Finally, a particular interesting case must be explained. As Figure 4 shows, the seat–vote difference for the case of the USA (1998) is rather small (2.1 per cent) and the electoral system used to select the members of the House of Representatives is FPTP. Given this institutional setting, one should expect values closer to those found for the UK rather than to the values produced by a more proportional electoral system such as that used in Finland in 1999, as actually happens. The explanation for this lies in the dual party system and the practically even distribution of popular support between Republicans and Democrats during the 1998 legislative elections.

All these examples show, then, how the minimum share of the total vote produced by aggregated threshold functions, despite being rather

unlikely in real life, does anticipate the trends that will be found when elections actually occur in the electoral systems under consideration. In summary, aggregated threshold functions anticipate and describe mechanical patterns of electoral systems but do not predict electoral outcomes.

Conclusion

More than three decades after the publication of Loosemore and Hanby's (1971) famous index of proportionality, the debate on how to measure electoral systems is not over (Taagepera 2007). The disproportionality index created by these scholars was the starting point for finding the best way to measure how votes are converted into seats. This article contributes to the debate by explaining how aggregated threshold functions summarise in a unique value the mechanical functioning of any electoral system by taking into account all of its institutional components. The values obtained from these functions have three properties that may attract the attention of both electoral reformers and students of electoral systems.

First, aggregated threshold functions are independent of electoral results. As opposed to most of the indices derived by Loosemore and Hanby, or Gallagher, the values obtained with these functions do not need any previous distribution of votes and seats to work. Aggregated threshold functions can calculate the necessary and sufficient conditions for winning any given number of seats just by using the institutional variables that define complete electoral systems. Electoral reformers can benefit from having such a straightforward tool to help forecast some of the political consequences of new electoral systems. Aggregated threshold functions show the effects of a variation in the number of districts, the assembly size or the electoral formula on the entry of minor parties or the probability of producing single-party cabinets. Reformers can anticipate that if a new system is far from the line of perfect proportionality, the probability of having a single-party cabinet will increase but the voice of minor parties may decrease, and that may have some implications if the number of cleavages is high. On the other hand, reformers will also be able to know in advance that systems closer to perfect proportionality will increase the probability of generating a divided government but will probably mirror society more accurately.

The second and the third properties that characterise these functions are quasi-universality and generality. The former means that aggregated threshold functions can specifically be applied to a broad range of complete electoral systems taking into account just the institutional components that define them: electoral formula, number of districts, assembly size and distribution of district magnitudes. In this article, aggregated threshold functions have been applied to 142 different complete electoral systems that have been used in 525 democratic elections between 1946 and 2000.

Generality means that the values obtained from the functions reflect the mechanical behaviour of the electoral system as a whole.

These two properties can appeal to students of electoral systems because the information they convey represents an accurate picture of how the electoral system behaves from a strict mechanical point of view. Since the values produced by ATFs are continuous, they are also significantly richer than categorical variables that characterise electoral systems based on one distinctive feature such as the electoral formula. Scholars working on electoral systems can now benefit from a measure that explains how close or how far the complete electoral system under study is from perfect proportionality.

This way of characterising complete electoral systems provides some initial interesting conclusions. For instance, when *single-seat district* electoral systems are made of single-member districts, aggregated threshold function results only depend on the number of competing parties. In this sense, the greater the number of parties, the lower the minimum share of the vote required to win half of the seats in parliament. In the case of PR, quota-based electoral systems produce, on average, results closer to the line of perfect proportionality than divisor-based systems. The values obtained when aggregated threshold functions are applied to half of the seats in the parliament also reveal that the number of districts is an important variable. If all defining variables except the number of districts are held constant, the minimum value necessary to win half of the seats in the legislative assembly decreases as the number of districts increases.

Aggregated threshold functions provide students of electoral systems with useful information that not only improves the information that has been produced by existing proportionality indices but also allows understanding of the functioning of electoral systems as a whole. We now have a measure that allows us to understand the relationship of electoral systems with other political phenomena. If it is argued that centre-left governments are more likely to appear in PR electoral systems (Iversen and Soskice 2002), we can now be more precise and establish under what specific institutional designs this type of government is more common. We can also use aggregated threshold functions to understand under which particular electoral system we should expect higher levels of corruption (Chang and Golden 2007) or whether deviation from perfect proportionality explains the formation of single-party governments or coalition governments. These are the type of research questions that the values generated by aggregated threshold functions may help answer.

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Notes

1. However, as mentioned by Ruiz-Rufino (2007), ATF are not electoral oracles. These functions do not anticipate the performance of any political party. This performance, i.e. the share of votes and the corresponding seats, will be determined by the distribution of votes that is obtained after the occurrence of each election. While taking this clarification into account, the trends anticipated by the values generated by aggregated threshold functions match the actual electoral results that can be observed. See below.
2. All data used in this article are available at <http://www.march.es/webpages/r.ruiz-rufino> and also upon request.
3. In this article, by mechanical behaviour or mechanical effect is meant the transformation of votes into seats given the institutional components of an electoral system.
4. A note of caution is necessary here. As I have pointed out, the main purpose of this article is to measure the mechanical behaviour of any electoral system using two particular ATF values. Thus, other important aspects of electoral systems like the psychological effects that they may produce in the voters are not taken into consideration (Duverger 1954). The inclusion of such effects is beyond the scope of this research. The main goal of this article is to introduce a consistent method for measuring the functioning of the different institutional variables that form an electoral system without taking into consideration any rational calculation made by the voters.
5. For some important scholars the type of ballot is also a key element of an electoral system (Rae 1971; Lijphart 1994). For instance, Rae hypothesised that the type of ballot could explain the fractionalisation of the party system and Lijphart offered one of the first empirical tests of this idea. I agree that the ballot is important but its distinction between open and close ballots incorporates some discretion on the voter's side that is hard to quantify and makes it difficult to include it in the ATFs.
6. A full account of the formal definition of aggregated threshold functions can be read in Ruiz-Rufino (2007).
7. This property also allows a novel approach to the study of electoral systems, since the most influential existing studies in the field (Rokkan 1968; Rae *et al.* 1971; Taagepera and Shugart 1989; Lijphart 1994; Katz 1997) are focused on the district level. There are, nonetheless, a few references in the literature that attempt to calculate thresholds nationwide (Taagepera 1998, 2002). However, these studies suffer from some limitations: they only focus on a very limited number of electoral formulas.
8. As I will point out below, ATF cannot be applied to those electoral systems that use open ballots. The reason is that it is hard to quantify the different electoral preferences that each voter marks in the ballot.
9. For a full development of this predictor see Taagepera (2007: 133–4).
10. Average district magnitude is calculated as follows:

$$\hat{M} = \frac{S}{E}$$

where E refers to the number of districts in which the territory is divided (see Lijphart 1994).

11. Ruiz-Rufino used the effective number of competing parties (ENP) (Laakso and Taagepera 1979) when testing the capacity of prediction of aggregated threshold functions. ENP was used as a proxy for the number of competing parties with a real chance of winning a seat in the legislature. The proxy worked well empirically. However a legitimate and sound

criticism against the use of such a predictor could be raised because the ENP depends, by definition, on election results produced by the electoral system in question.

By using the new indicator suggested by Rein Taagepera, this criticism is no longer valid. Using the same data provided by Ruiz-Rufino (2007), I have re-tested the capacity of predictability of aggregated threshold functions using Taagepera's new predictor of seat-winning parties instead of ENP and similar satisfactory results have been obtained. Therefore, as far as aggregated threshold functions are concerned, the new indicator calculated by Taagepera does seem to be a good predictor of the number of parties with a real chance of winning a seat in the legislature.

12. Taagepera explains that what this predictor offers is the geometric mean of the number of seat-winning parties. By using this particular average, the variation in the number of seat-winning parties across elections is already incorporated.
13. By necessary share of the total vote is meant the proportion of votes counted nationwide that a party must win to obtain a certain number of seats. In other words, let S_T be a given number of seats somehow distributed among all districts in which the country is divided; similarly, let $V_{S_T}^{nec}$ be the total share of the vote necessary to win those S_T seats. If a party fails to win a total share of the vote greater or equal to $V_{S_T}^{nec}$, then, it cannot get S_T seats.

Sufficient share of the total vote refers to the proportion of votes counted nationwide that suffices to win a determined number of seats. Following a similar notation, $V_{S_T}^{suf}$ defines the total share of the votes that suffices to win a given number of seats in the legislative assembly, so, if a party wins a share of the vote greater or equal to $V_{S_T}^{suf}$, then that party will get at least S_T seats. Conceptually, these ideas are the same as the substantive meanings of threshold of inclusion (Rokkan 1968) and threshold of exclusion (Rae *et al.* 1971).

14. Some complete electoral systems establish a minimum share of the total vote in order to win political representation. These legal thresholds (T_L) are largely intended to prevent the entry of minority parties into parliament. This variable is, obviously, of some importance, although this article does not deal with it. However, the way to proceed is the following: if $T_L > V_{S_T}^{nec}$ then T_L gives us the share of the votes that are required to win the given number of votes for the which aggregated threshold function is calculated ($V_{S_T}^{nec}$).
15. An electoral system is said to be perfectly proportional if the share of votes obtained by any given party gives it the same share of the seats in the legislative assembly. Formally, for any political party p , $V_p = S_p$ where V_p stands for the share of the total vote won by party p , and S_p stands for the share of total seats in the parliament assigned to party p .
16. Electoral systems that use the single transferable vote or alternative vote, for example, cannot be characterised here given their strong dependency on a previous distribution of votes that aggregated threshold functions do not consider. The electoral systems excluded are limited vote (LV), single non-transferable vote (SNTV), Borda, alternative vote (AV) and single transferable vote (STV). Although these electoral systems include those used in Ireland and Japan, the number of elections that occurred under all these electoral systems is small.
17. Practically, all the remaining elections have taken place under some type of two-tier electoral system. As a matter of fact, aggregated threshold functions can also be applied to some of these mixed and multi-tier electoral systems. However, given the extraordinary complexity of these electoral systems, I prefer to treat this issue in a separate article. The values for these electoral systems are, however, available upon request.
18. A similar reasoning applies for half of the seats in the legislative assembly. Consider the following example. Take a complete electoral system where 100 seats are elected in the parliament using the Hare quota. These seats are distributed in eight districts according to the following distribution $M_d = [50 \ 15 \ 15 \ 4 \ 4 \ 4 \ 4 \ 4]$, where M_d indicates the vector that contains all district magnitudes. Given this institutional setting, aggregated threshold functions are applied to the combination of seats that produces the minimum value needed to win a given number of seats. Since we are interested in half of the seats in parliament, the combination that produces such a value would be $S_d = [1 \ 14 \ 15 \ 4 \ 4 \ 4 \ 4 \ 4]$, where S_d indicates the vector that shows all seats won in each of the districts contained in M_d . For divisor-based electoral systems that use the d'Hondt electoral formula, the

combination of seats that minimise the function for a total number of seats equal to half of the parliament is $S_d = [0 \ 15 \ 15 \ 4 \ 4 \ 4 \ 4]$. Note how in this case no seats are won in the biggest district.

19. This is so, though, only for half of the seats in Parliament. For simplicity, and in order to ease the replication of data, the values presented in this article for winning a majority in the parliament have been calculated using the average district magnitude.
20. This strong restriction requires some clarification. Run-off electoral systems like TRS may invite voters to behave strategically and the overall mechanics of these systems may not be completely captured by aggregated threshold functions. While accepting this, I should point out that aggregated threshold functions only account for the mechanical effects of electoral systems and not for the psychological effect that they may produce. On this ground, I consider it defensible to characterise TRS by just using the first round since the second round would only apply in the case that no party obtained a majority of the votes. Aggregated threshold functions would still show, in any case, the minimum share of the total vote required to win the number of seats at stake.
21. The appendix shows the mathematical expression that has been used in this case.
22. By definition, a *sine qua non* condition for a country to qualify as a democracy is that it holds competitive elections (Przeworski 1991). This means that at least two political parties must take part. Given that the number of parties is calculated here depending on the size of the assembly and average district magnitude, only those countries whose assembly is bigger than 16 produce two parties with a real chance of winning seats. Figures below this may be interpreted not as meaning that the country is undemocratic, but that it produces a clear winner and a clear loser. Only three countries generated fewer than two parties: St. Kitts, St. Vincent and Grenade.
23. This finding confirms mathematical and empirically a remark already made by J.S. Mill (1991).
24. Obviously, the complete electoral system used in St. Kitts produced the highest value needed to win half of the seats since this electoral system produced the smallest number of competing parties in the sample. The complete electoral system that produced the smallest value for winning half of the seats in the parliament was that used in the United Kingdom from 1950 onwards. According to the predictor proposed by Taagepera, about five competing seat-winning parties were produced by that complete electoral system.
25. For a more detailed account of the working and formulation of quota-based electoral formulae see Taagepera and Shugart (1989) and also Penades (2000).
26. For example, the legal threshold for winning representation under the Moldovan electoral system is 4 per cent and the aggregated threshold value is about 0.91 per cent. In this case, the minimum share of the total vote needed to win a seat is equivalent to the legal threshold. When legal thresholds apply not at national level but at district level instead, then aggregated threshold functions do usually show the total share of the vote necessary to win a seat. In Spain, for example, a legal threshold of 3 per cent at district level is required to win representation; since Spain has two uninominal districts, aggregated threshold functions take the value of either of these districts as the minimum necessary to win a seat. Given the institutional components of the Spanish electoral system, the minimum value to win a seat in either of these districts is 12.5 per cent, which is much higher than the legal threshold.
27. The maximum value for a quota-based system is that for the one used in Israel from 1951 to 1969. In the case of divisor-based electoral systems, the maximum is for that in use in the Netherlands from 1946 to 1998.
28. Only Luxembourg and the Slovak Republic used a different quota, namely, the Droop quota.
29. A good example is the electoral system in used in Chile since 1993. There, 120 seats are distributed in 60 equal districts, so producing two-seat districts. When district magnitude is that small, proportional representation formulae play virtually no role.
30. The so-called 'new state' paradox explains the variation of the Loosemore and Hanby's index when a new party enters the competition winning a seat and the existing parties maintain their vote share. Illustrations of the paradox are offered by Gallagher (1991: 39) and also by Taagepera (2007: 79–82).

31. Loosemore and Hanby's index, Rae's index and Gallagher's least-squares index vary from 0 (perfect proportionality) to 1 (full disproportionality). The Sainte-Laguë varies from 0 (perfect proportionality) to ∞ (full disproportionality).
32. In fact, as both Figures 3 and 4 show, these two countries are closer to the 25 per cent line than to the point of perfect proportionality.

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APPENDIX

Aggregated Threshold Functions for *Single-Seat District* Electoral Systems

1 Seat

The aggregated threshold function applied to those *single-seat district* electoral systems referred to above has the following form:

$$V_{S_T}^{nec} = \frac{M_d}{S} \left(\frac{S_d - 1 + c}{M_d - 1 + Pc} \right) * S_T \quad (3)$$

When $M_d = S_d = 1$ and $S_T = 1$, then

$$M_d = \frac{1}{SP} \quad (4)$$

where M_d stands for district magnitude of a given district; S_d stands for the number of seats won in a given district; S_T stands for the total number of

seats for which aggregated threshold functions are applied; S stands for the number of seats elected in the legislative assembly; P stands for the number of seat-winning parties as calculated by Taagepera (2007) and c stands for an adjustment term as defined below.

Half of the Seats in the Parliament

Since $S_T = \frac{S}{2}$ and $M_d = S_d$, then according to function 3 the form of ATF is,

$$V_{S_T}^{nec} = \frac{M_d(M_d - 1 + c)}{2(M_d - 1 + Pc)}, \quad (5)$$

When $M_d = 1$, function 5 can be simplified as,

$$V_{S_T}^{nec} = \frac{1}{2P} \quad (6)$$

Aggregated Threshold Functions for Divisors-based Electoral Systems

The aggregated threshold function for divisor-based electoral systems has the following form:

$$V_{S_T}^{nec} = \sum_{d=1}^E \frac{M_d}{S} \left(\frac{S_d - 1 + c}{M_d - 1 + Pc} \right) \quad (7)$$

where M_d , S_d , S_T , S and P are already defined. E stands for the number of districts and c stands for the adjustment term.¹

Aggregated Threshold Functions for Quota-based Electoral Systems

The aggregated threshold function for quota-based electoral systems has the following form:

$$V_{S_T}^{nec} = \sum_{d=1}^E \frac{M_d[P(S_d - 1) + 1 + n]}{SP(M_d + n)} \quad (8)$$

where M_d , S_d , S_T , S , P and E are already defined and n stands for the modifier of the quota² (Taagepera and Shugart 1989).

How to Calculate ATF when the Distribution of District Magnitudes is Missing

Sometimes, the vector containing all district magnitudes is missing. In those cases, we can calculate a proxy that generates closer results with the actual ATF.³ These predictors are also limited to calculate the values equivalent to half of the seats in the parliament. This proxy is based on the concept of

average district magnitude and average seats. Average district magnitude is defined as

$$\hat{M} = \frac{S}{E} \quad (9)$$

Average seat is defined as

$$\hat{S} = \frac{\hat{M}S_T}{S} \quad (10)$$

When information about the distribution of seats among all districts is missing, ATF for divisor-based electoral systems can be calculated using the following proxy function

$$\hat{V}_{S_T}^{nec} = \frac{\hat{S} - 1 + c}{\hat{M} - 1 + Pc} \quad (11)$$

If $S_T = \frac{S}{2}$, then from expressions 9, 10 and 11 we obtain the following,

$$\hat{V}_{S_T}^{nec} = \frac{S + 2E(c - 1)}{2[M + E(Pc - 1)]} \quad (12)$$

If the complete electoral system use a quota-based electoral formula, then, again based on expressions 9 and 10, the following proxy function can be defined for those cases when the distribution of district magnitudes is not known:

$$\hat{V}_{S_T}^{nec} = \frac{P(\hat{S} - 1) + 1 + n}{P(\hat{M} + n)} \quad (13)$$

If $S_T = \frac{S}{2}$, then from expressions 9, 10 and 13 we obtain the following,

$$\hat{V}_{S_T}^{nec} = \frac{P(S - 2E) + 2E(1 - n)}{2P(S + En)} \quad (14)$$

Notes

1. The parameter c refers to the adjustment term that is needed to allocate seats according to a constant non-negative criterion. When $c = 1$, then the d'Hondt formula is defined. When $c = 0.5$, then the Sainte-Laguë is defined. A full development of this idea can be found in Balinski and Young (1982; see also Penadés 2000).
2. The quota is a ratio obtained dividing the total number of votes by the district magnitude. When $n = 0$, then the Hare quota is defined. If $n = 1$, then the Droop quota is defined.
3. The correlation between the values generated by ATF and the values generated by this proxy is 0.85.